

### Quantum theory appendix 3: it from qbit

The very simplest instance of information in the classical world is a bit-it is either on or off, zero or one, YES or NO. The quantum mechanical analogue is a qbit. This can also be in a superposition of YES and NO. Say 20% YES and 80% NO, or 50% YES and 50 % NO. The interesting thing is that there are signs that matter, so the following two states are distinct:

50% YES + 50% NO  
50% YES - 50% NO

The meaning of the plus and minus sign is not intuitive, but it is easy to explain. The idea is that the answer to only one question can be coded into a bit-YES or NO. But the answers to two questions may be coded into a qbit ie the states YES and NO might code the answer to the question, Do you love her (or him)?. Let us call these states of definite love-either LOVE or NOT LOVE. The superpositions of definite love states can be used to code the answer to a different question, say, did I wear WHITE or BLACK tonight? So the coding is as follows:

BLACK:                50% YES + 50% NO  
WHITE:               50% YES - 50% NO

Note that when you are in a definite colour state your state of love is indefinite. This is the way life is in the quantum world. Get used to it, for it gets stranger.

Now let us play a quantum dating game. Alice and Bob are introduced by me and have a date. After that they go home (separately) and each calls me to report on the date. Anyone who has tried to be a matchmaker is familiar with what can happen.

Bob says he loves Alice, but Alice does not love Bob. We can represent this by a state

IBOB=YES, ALICE=NO >

The funny notation is one that is used for quantum states.

Or the opposite can happen

IBOB=NO ALICE=YES >

But in quantum mechanics something very peculiar can happen which is that they can together be in a state where there is a hundred percent certainty that they disagree, while the chance that each will give either answer is even. This is a state I will call

ICONTRARY>

In the contrary state, the chances that Bob loves Alice, when asked is 50%. The chance that Alice loves Bob is 50%. But the chance they disagree is 100%. That is, if Bob loves Alice then we can be sure that Alice does not love Bob. And if Bob does not love Alice then Alice will be smitten.

It turns out that this state can be expressed as a superposition of the two states where they know who they love:

$$| \text{CONTRARY} \rangle = | \text{BOB}=\text{YES}, \text{ALICE}=\text{NO} \rangle - | \text{BOB}=\text{NO}, \text{ALICE}=\text{YES} \rangle$$

We say that Bob and Alice are entangled. Each of them is in an indefinite state, but together they share a property, that of contrariness.

Now suppose that you think that quantum mechanics is an approximation to a deeper theory. Then maybe the mysterious correlation can be explained. For example, suppose that Alice and Bob hit it off and decided to play a trick on me. As soon as I ask Alice if she loves Bob, she picks an answer randomly, but she texts Bob right away, so he knows what Alice has responded, and answers the opposite.

This would be called a hidden variables theory. And it would be a local theory that used information being transmitted between the two actors at less than the speed of light. So there would be nothing strange about that.

It turns out that in quantum theory you can check if there is such a channel of information being used. A remarkable theoretical result by John Bell, together with some clever experimental work, showed that this is impossible. Unless information is transmitted instantly, not just at the speed of light, but immediately, you can do a version of this experiment where the result will disagree with the predictions of quantum mechanics.

Let me give you a rough idea of how this works. It uses the fact that there are other questions I could have asked the young couple, like what colour did they wear on the date. Recall that if their color state is definite then their love state is indefinite, or vice versa. But what is interesting is that they can still have entangled color states.

Asking them both about colour rather than love, we can imagine states like  $| \text{BOB}=\text{BLACK}, \text{ALICE}=\text{BLACK} \rangle$  which I expected as they both live in New York. Or the answers might have been

$$| \text{BOB}=\text{BLACK}, \text{ALICE}=\text{WHITE} \rangle$$

or

$$| \text{BOB}=\text{WHITE}, \text{ALICE}=\text{BLACK} \rangle$$

This way they intrigued each other. But they also might have been in the contrary state in which case, the situation for colour is the same as for love. For it turns out that the same state |CONTRARY> we had above, can be expressed as

|CONTRARY> = |BOB=BLACK, ALICE=WHITE > - |BOB=WHITE, ALICE=BLACK >

That is, the contrary state has them contrary on all questions. As individuals they are in completely indefinite states, they might be in love or not, with 50% probability. They might have worn black or white, with 50% probability. But if I ask them both about what they wore, then I can be sure of getting contrary replies for colour. If I ask them about love, the same thing.

Indeed, there are an infinite number of questions I might ask, for example, there is a question in the quantum world that corresponds to 20% being black and 80% being in love. This doesn't mean anything in the classical world but there are definite states of qbits that correspond to a definite answer-YES or NO-to this question.

It turns out that whatever the question is-so long as I ask the same question to both parties, I will get contrary results. While if I ask either one singly any of the questions the answer will be YES half the time and NO half the time.

Bell's theorem is very simple. He allows there to be any amount of information hidden in Bob and Alice individually. He makes only one assumption, which is that the answers that Bob gives to the question he is asked is not affected by the choice of which question Alice is asked.

Bob and Alice may have conspired during their date. They may have adopted an intricate, subtle strategy. Bell assumes only that their strategy cannot involve Alice telling Bob what question I ask her, or vice versa.

Once this is assumed then Bell proves that the probabilities of different outcomes must differ from those predicted by quantum mechanics.

We can call a hidden variables theory local if it does not let Bob and Alice communicate once their date is over and they go home to call me. Bell's theorem establishes that no local hidden variables theory can agree with the predictions of quantum mechanics.

Of course it was then an experimental question: maybe quantum mechanics is wrong and local hidden variables theories are right. The experiments come down definitely against local hidden variables and in favor of quantum mechanics.

There are two possible conclusions. Quantum mechanics is the final theory, there is no deeper level of description. Or there is a deeper theory behind quantum mechanics, but it is non local in a very strange sense, which is that once two people date, they are in a state where from then on, a question asked to one will influence the state of the other, no matter how far apart they may have separated.

Applied to particles this means that when two particles interact, they become entangled, such that even if they separate they share properties. After that, the response of one of the particles to a further interaction can depend on what the other particle next interacted with.