Relativity appendix 1: The relativity of simultaneity and its consequences

That idea of the relativity of simultaneity is not difficult to explain if we stick to the operational philosophy. The key point is that if we want to talk about whether two events distant from each other take place at the same time, we must construct two clocks and describe how to synchronize them. For our purposes, a clock is simply a small device with a face that shows a number, and each time you look at it the number is bigger. So by a clock we really mean a combination of clock and calendar—something that, say, measures the number of seconds since some notable event like the Beatles landing in New York.

The reader may worry about whether the clock is good enough. Is it ticking regularly? We apply the operational philosophy here also. We define a good clock as one whose measurements conform to Newton's laws of motion. In particular, we can test whether Newton's first law is satisfied. That law, also called the principle of inertia, asserts that a body with no forces acting on it moves equal distances in equal times. We can check this by recording the motion of a ball rolling on a table. If it covers the same distance on the table in each equal increment of time measured by our clock, then that clock is working well.

This would seem to make Newton's law a matter of defining time. If we observe the motion of only one ball, Newton's law is a definition of time. But once we have used one rolling ball to check our clock, we can use that clock to check the law on many other sorts of moving bodies. The fact that the law works no matter how many bodies are tested is a measure of its effectiveness.

So far, this is just Newtonian physics from an intelligent perspective. Life gets interesting when we seek to attribute times to events far away from us and our clock. Let's imagine that aliens live on a planet in orbit around Proxima Centauri, the nearest star. It's natural to wonder what they're doing right now. To investigate that operationally, we need to communicate with them. We send radio signals, which take four years to travel each way. So that we and the aliens can have a discussion about time, we ask them to build a clock identical to ours. To establish which events at Proxima Centauri are simultaneous with events on Earth, we need to synchronize their clock with ours. This sounds easy, but it will turn out that however we do it there is an ambiguity that leads different observers to disagree with each other.

Let's consider two specific events: midnight on New Year's Eve in Toronto, which we'll call A, and some particular event at Proxima Centauri, which we'll call B. In the most obvious test of whether these two events at different places are simultaneous, we put a third observer, Ralph, halfway between Earth and Proxima Centauri. Since he's the same distance from each, we can reason as follows:

When A happens, Earthlings send a radio signal to Ralph. When B happens, the aliens also send a radio signal to Ralph. It seems logical to assert that if Ralph receives both radio signals at the same time, then the events took place at the same time.

There's a problem, however, with this test. Ralph could be moving, and this will mess the test up. He could be accelerating, in which case the result could be almost anything. But Ralph can tell if he's accelerating, because if he is, he will feel its effects, so let's assume that Ralph is not accelerating but has a constant velocity. According to the principles of special relativity, motion with constant velocity is relative, so he cannot experience or detect his own motion. But the test we're discussing must be formulated entirely in terms of what Ralph can observe. So we can't ask any more of him during the test than not to accelerate. This leaves the possibility that he is moving with respect to Earth. Let's imagine, for example, that he's moving toward Earth.

And let's imagine that he has a brother, Randy, who stayed halfway between Earth and Proxima Centauri and did receive the two signals at the same time. If Randy gets the two signals at the same time, Ralph won't, because he's moving toward the signal coming from Earth and away from the signal sent by the aliens on the planet orbiting Proxima Centauri. So he will receive the Earthlings' signal before that of the Proxima Centaurians.

So we have a disagreement: Randy judges A and B as passing the test that indicates they were simultaneous, while Ralph does not.

We could decide that Randy was right, because he stayed halfway between Earth and Proxima Centuri. But this assumes that Earth and the planet orbiting Proxima Centauri are not moving with respect to each other. Suppose that's not true. Suppose that Proxima Centauri is moving toward Earth. Then it could be Ralph who stays halfway between them.

Moreover, the events need not be associated with something that continues to move. Suppose that event B is the collision of two asteroids near Proxima Centauri. The asteroids are moving with respect to both Earth and Proxima Centauri, but it doesn't matter. They either collide or don't collide—if they do, that is the event, period. To eliminate concern about the motion of Earth or the planet orbiting Proxima Centauri, or any other body, we will elaborate on our test of the simultaneity of two events.

This time we start with Ralph sending two signals at the same time, one toward Earth and one toward Proxima Centauri. (We assume it is unproblematic to say that two events take place "at the same time," as long as they're at the same place.) We will stipulate that event A occurs when Earth gets the signal and event B occurs when the Proxima Centaurians get their signal. Signals are then sent from A and B immediately back to Ralph. The test will declare events A and B simultaneous—*according to Ralph*—if signals immediately reflected back from those events reach Ralph at the same time.

Setting up the test this way eliminates the need to check whether Ralph is halfway between Earth and Proxima Centauri. This is covered by the condition that two signals sent initally by Ralph arrive at each event.

You might worry that the test depends on how the devices at A and B that reflect the signals back are moving. But this is not the case. The reason is a key property of electromagnetic radiation, which is that the speed of a photon is the same no matter how the source that emits the photon is moving. In fact, the speed of light is, as far as we know, completely universal. Two photons emitted together—say, from a supernova—travel together, no matter whether they have the same energy or color or frequency and no matter how their source was moving. They travel at the universal speed of light. So the definition we have given depends on nothing but the two events and the observer, Ralph.

I hope it is clear that this second definition includes everything we could mean by two distant events happening at the same time. It depends on the observer, but any observer who tried this second test on two events would be satisfied to declare the distant events simultaneous if they passed the test.

This was the test for simultaneity of distant events that Einstein used in his first paper on special relativity.[1] The historian Peter Galison has made the case that Einstein learned about this test from his work in the patent office, where one speciality of his was judging patents related to telegraphy. According to Galison, some of the inventions Einstein would have examined were intended to establish the simultaneity of distant events by the exchange of telegraph messages.[2] The synchronization of two distant clocks was a problem the railroad companies had to solve if the trains were to be run safely and efficiently.

With a bit more thought one can argue that there is no definition of distant simultaneity better than Einstein's. The reader is invited to try inventing better tests.

Once we're happy with our operational test for distant simultaneity, we can ask whether or not it gives unambiguous answers. As I defined it, the answers depend on a particular observer, Ralph, who sends and receives the radio transmissions. What if two separate observers apply the same test. Will they agree about the results?

The answer is that the results of the test depend on how an observer is moving. Let us return to the case of two observers, Ralph and Randy, who are moving with respect to each other (see Figure **13)**. Let's assume they coincide at the first step of the test, when they send photons out in opposite directions, to Earth and to Proxima Centauri. It's easy to see that if A and B satisfy the test for being simultaneous with respect to Ralph,

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[1] "Zur Elektrodynamik bewegter Körper." *Annalen der Physik* 17:10, 891–921 (1905).

[2] Peter Galison, *Einstein's Clocks, Poincaré's Maps: Empires of Time* (New York: W. W. Norton, 2003).

they will fail it with respect to Randy. This is because the fact that Randy is moving toward Earth with respect to Ralph means that if the two reflected photons get to Ralph at the same time, the one from Earth will reach Randy before the one from Proxima Centauri.

However, there are other pairs of events that Randy will see as passing the test for simultaneity. Let's call two of them C and D. It's easy to see that, for the reason just discussed, Ralph will see them as failing the test, because the photon from D at Proxima Centauri will reach him before the photon from C on Earth.

Recall that we were careful to specify that neither of these two observers is accelerating; they are each moving with a constant speed and direction. If the principle of relativity is true, then there's no way to decide whether both of them are moving or one (and which one) is moving and the other is at rest. The only meaningful thing to say about their motion is that each is moving with respect to the other.

This is a symmetric relation. Ralph is moving with respect to Randy and Randy is moving with respect to Ralph. Neither is, in any objective sense, at rest, for there is no absolute meaning to being at rest. Hence there is no known objective test to determine whose point of view is more objective, or more correct.

Note that the relativity of observers moving with a constant speed and direction—also called the relativity of inertial frames—is not a philosophical principle . It is a contingent fact, which is established by observations. We cannot rule out the possibility that a new physical phenomenon will at some point be discovered that will enable us to distinguish motion from rest and break the symmetry between Ralph and Randy. Up until now, the principle of the relativity of inertial frames has been well tested in a diverse range of experiments and observations, from the scale of elementary particles to the scale of galaxies. It has been tested in the physics of the highest-energy cosmic rays—protons traveling with an energy equivalent to a hundred billion times their mass—which travel hundreds of millions of light-years to reach us. This gives us a test of special relativity to one part in a hundred billion. The principle that photons travel with a universal speed has been tested by studying photons created in a sudden burst called a gamma-ray burst, caused by two neutron stars colliding ten billion years ago. The photons start off together, racing across the universe, and arrive ten billion years later at a satellite in Earth orbit. There is no evidence that photons with different energies arrive at different times.

It is possible that future experiments will lead to the discovery of a breakdown of the principle of the relativity of inertial frames, but until that happens, this is among the best-established scientific principles we know. But the relativity of simultaneity is a direct consequence of special relativity, so we should take it very seriously.

The relativity of simultaneity is a big step in the expulsion of time from the physicists' conception of nature, implying, as it does, that there is nothing real in the universe corresponding to the experience we have of time passing.

We are used to thinking that other people experience time passing as we do. Indeed, if time passing is something real, then every observer must experience it. We naturally believe that our experience of a moment is shared by others. But if it is, then shouldn't it be shared by all observers, wherever they are? Unfortunately, if the relativity of simultaneity is right, then there can be nothing about our experience of time passing that is shared with distant observers.

Let's see why.   Frances, as an observer, has an experience of time passing, of reality developing through a series of present moments. If this is not just a subjective illusion, then there is something real in nature corresponding to her experience of the present moment. If this is the case, this "something real" should be something all observers can experience and will agree about, wherever they are in space and however they are moving.   Let us fix a moment for Frances—say, noon on January 4, 2011. An observer, no matter where he is and how he is moving, should be able to identify the reality he observes that is simultaneous with what Frances experiences at that moment.

As it happened, Frances was on a train at noon on January 4, 2011, which was being passed at that moment by a train going in the opposite direction. Through the window, she glimpsed a beautiful man on the other train, Daniel, and instantly fell in love with him. In this rare case, the experience was mutual. At that same moment, Daniel saw Frances through his window and was also taken with her. A month later, they met on the train platform, and the rest, as we say, is history.

Daniel and Frances shared an experience of a moment, which is evidence that there could be something real corresponding to the human experience of a moment of time. But this is not yet sufficient evidence. If that moment of time corresponds to something real in nature, then it must correspond to something real for every observer.

The problem arises when we ask what is real in the observations of observers distant from the event. Let's consider the alien observer at Proxima Centauri. Is there something real that it observes corresponding to the moment of Frances's and Daniel's glimpse of each other?

The answer is no. The reason is that because of the relativity of simultaneity there is a whole range of events at Proxima Centauri that can be considered simultaneous with Daniel's and Frances's fleeting meeting. Frances has a clock and can use the procedure we outlined above to synchronize a clock at Proxima Centauri with hers. According to that definition of simultaneity, there is a particular event at Proxima Centauri simultaneous with her glimpsing Daniel. Let us call it X.   X might be a proxima-cat waking up, but it doesn't matter.   What does matter is that Daniel can carry out the same procedure, and it will lead to the identification of a different event—event Y—at Proxima Centauri.

Depending on which of them is traveling toward Proxima Centauri, the events X and Y will occur in different orders. Suppose it is Frances who is traveling toward Proxima

Centauri; then X will occur before Y. X might even be the cause of Y. For example, X might be an alien e-mailing a friend, who then reads the message at Y. So both X and Y cannot be real at the same time. But Frances says that X is simultaneous with the trains passing, and Daniel says that it is Y that is simultaneous with the train passing. They cannot both be right, but according to the principle of relativity of inertial frames, there is no reason to prefer one story over another. So we must conclude that "what is simultaneous with what" corresponds to nothing real; it is only a convention that depends on the motion of particular observers. This means that there is nothing real at Proxima Centauri corresponding to the moment of time when the two trains passed on Earth.

We do not have to limit consideration to just Daniel's and Frances's definitions of simultaneity. Any observers who might have seen the two trains passing will have their own own opinion as to which event at Proxima Centauri is simultaneous with the trains passing on Earth. They will disagree, and by a lot. If we include every possible observer, the times they attribute to events **at** ~~on~~ Alpha Cent**a**uri simultaneous with the passing of the trains on Earth can differ by as much as eight years.

Hence the only meaning that can be given to asserting that two events at different places are simultaneous depends on a particular observer. Different observers will disagree with one another. But if something is real, it must correspond to something all observers will agree about. So there can be nothing real at Proxima Centauri corresponding to Frances's and Daniel's glimpse of each other. Nor is there anything in nature that is objectively real that corresponds to their experience of that moment of time.

It gets even worse than that. Suppose we had a reason to break the symmetry and assert that X is really simultaneous with the trains passing. Then we can find an observer at Proxima Centauri, moving toward Earth, who sees that X is simultaneous not with the two trains passing but with an earlier event, when Frances bought her ticket for the train. But "being simultaneous with" should be what philosophers call a transitive relation. Consider three events: A, B, and C. If A is simultaneous with B and B is simultaneous with C, then A is simultaneous with C. But we have just shown that the event X at Proxima Centauri is simultaneous with two different events on Earth—one when Frances buys her ticket and one when her train passes Daniel's train. But those two events can't be simultaneous, because she can ride the train only after she buys the ticket.

Hence if the relativity of simultaneity and the relativity of inertial frames are both true, there cannot be anything physically real corresponding to the intuitive idea of A happening at the same time as X, if A and X happen in different places.

The experience we each have of time passing, revealing a succession of present moments, then cannot correspond to anything real in the world. The sense that time passes must then be an illusion.

In what way could this illusion be generated? The theory of special relativity does not tell us, but its argument is strong nonetheless. The relativity of simultaneity leaves no room for a picture of the world in which time passing corresponds to a real phenomenon.

What is, then, real in the world as described by Einstein's theory of special relativity? The answer is, certain relations of causality. The event T—say, Frances buying her ticket—is a cause of the event P, which is her glimpse of Daniel as their trains pass. There must be something objectively real about T coming before P, because P could not occur had T not happened. This relationship of causality must be physically real in any sensible theory.

Indeed, special relativity allows for this, because all observers will agree, no matter which clock they use, that if one event is the cause of another event, the former always precedes the latter.

The event P has a large number of events that were partially causes of it—that is, events that had to happen for P to happen. Let's write down a list of all the causes of P. That list must correspond to something objectively true in nature. We call it the *causal past* of P. Similarly, the list of all events that P is partly a cause of must correspond to something real and hence must be something all observers will agree on. This is called the *causal future* of P.

If we could record the whole history of the universe, we could write out these lists of causal futures and pasts for every event in the history of the universe. This is called giving the *causal structure* of the universe. It is something all observers will agree on.

One thing to notice is that there are pairs of events that cannot be causes of each other. This is because the speed of light is the maximum speed by which a causal influence can travel. Consider P (the passing of the trains on Earth) and X, the event at Proxima Centauri simultaneous with it according to Frances's notion of time. Here is how Frances reasons. P cannot cause X, because it would take a signal traveling at infinite speed to reach X from P. Infinite speed, because speed is distance over time, and the time is zero.

Similarly, X cannot cause P. Moreover, that P cannot cause X and vice versa is something that all observers can agree on. Another observer may assert that according to *his* clock X happens after P, but that time interval will always be too short to allow anything traveling at less than the speed of light to reach X from P.